Hawking radiation from the cosmological horizon in a FRW universe

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It is well known that there is a Hawking radiation from the cosmological horizon of the de-sitter spacetime, and the de-sitter spacetime can be a special case of a FRW universe. Therefore, there may be a corresponding Hawking radiation in a FRW universe. Indeed, there have been several clues showing that there is a Hawking radiation from the apparent horizon of a FRW universe. In our paper, however, we find that the Hawking radiation may come from the cosmological horizon. Moreover, we also find that the Hawking radiation from the apparent horizon of a FRW universe in some previous works can be a special case in our result, and the condition is that the variation rate of cosmological horizon $\dot{r}_H$ is zero. Note that, this condition is also consistent with the underlying integrable condition in these works from the apparent horizon.

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I. INTRODUCTION

Since Hawking found that there was a thermal radiation like a black body in a black hole, it has been further found that the radiation is in fact due to the existence of event horizon [1]. The event horizon played a key point can also be seen from the Unruh effect where an uniformly accelerating observer with acceleration a in the Minkowskian spacetime can detect a thermal spectrum with temperature $T = a/2\pi$ [2]. Here the Unruh radiation is closely related to the existence of Rindler causal horizon for the observer. Obviously, the Hawking radiation with a temperature which is proportional to its surface gravity on the event horizon can give some insight on the deep relationship between gravity and thermodynamics. Indeed, the thermodynamics of black hole has already been constructed with the Bekenstein entropy of a black hole [3–6]. Note that, the Hawking radiation is usually investigated from the event horizon of a stationary black hole. In fact, it can also be obtained from the cosmological horizon of a spacetime such as the cosmological horizon of de Sitter spacetime [7, 8].

Event horizon and cosmological horizon are both global concepts [9, 10]. Strictly speaking, locally it is not known whether there is an event horizon or cosmological horizon associated with a certain dynamical spacetime at some time. Thus this causes the difficulty to discuss Hawking radiation for a dynamic spacetime. However, by using the the null property of event horizon or cosmological horizon and the intrinsic symmetry of a dynamic spacetime, we can find a corresponding hypersurface which can reduce the event horizon or cosmological horizon in the stationary case. Because of this, we also call this corresponding hypersurface as the event horizon or cosmological horizon for a dynamic spacetime in our paper [11–15]. In spite of that, another situation appears. This is, the event horizon (the above corresponding hypersurface) and apparent horizon for a dynamic spacetime are usually different, while they are consistent for a stationary spacetime. Therefore, the Hawking radiation from which horizon is still an open question. Recently, Hayward and other authors have attacked this question [15–17]. By using the quasi-local Misner-Sharp energy [18–20], the so-called unified first law can be deduced from the Einstein equation in a spherical symmetric spacetime [21–24]. And they argued that the Hawking radiation might come from the apparent horizon for a dynamic spherical symmetric black hole spacetime, since after projecting the unified first law on the apparent horizon of a dynamic spherical symmetric black hole spacetime, one can obtain an analogy of the first law of thermodynamics of stationary black hole. In addition,

one could use the Hamilton-Jacobi equation of particles to make a simple proof [17, 25]. However, there are other works showing that the Hawking radiation can come from the event horizon of dynamic black hole spacetime by investigating the behavior of the quantum filed near the event horizon [11–14].

On the other hand, the Friedmann-Robertson-Walker (FRW) universe is a dynamical spacetime, and the de Sitter spacetime can be its special case. Therefore, Hawking radiation may also exist in a FRW universe. By considering that the FRW universe is also a spherical symmetric spacetime and with an apparent horizon, therefore, the above discussion on the apparent horizon of dynamic spherical symmetric black hole spacetime can be generalized to the FRW universe. There have been many interesting works based on this issue [26–33], and it has been proved that the Hawking temperature of the apparent horizon in a FRW universe is $T = 1/2\pi r_A$, where the temperature is measured by the corresponding Kodama observer [34] and r_A is the radius of apparent horizon [31, 32]. In particular, note here that if we assume the entropy of apparent horizon S satisfying $S = A/4$, where A is the area of the apparent horizon, one is able to derive Friedmann equations of the FRW universe with any spatial curvature by applying the Clausius relation to apparent horizon [35, 36]. However, there is the same situation as the dynamic black hole spactime that the cosmological horizon of a FRW universe is not usually consistent with its apparent horizon. Therefore, one of our motivations is that which kind of results we will obtain if we investigate the behavior of the quantum filed near the cosmological horizon of FRW universe.

There are several methods to investigate the behavior of quantum filed near the horizon of a spacetime [37–39]. In our paper, we mainly use the Damour-Ruffini method first proposed by Damour and Ruffini and then developed by Sannan and Zhao [13, 14, 39, 40]. By using the fact that usually the Klein-Gordon equation in the tortoise coordinates can be reduced to the standard form of wave equation near the cosmological horizon of FRW universe, we obtain the appropriate parameter κ which corresponds to the surface gravity in the stationary case. Moreover, we find that the ingoing wave of FRW universe is not analytical on the cosmological horizon, and it can be extended by analytical continuation from the inside to outside of the cosmological horizon [13, 14, 39–41]. After doing these, we obtain the Hawking radiation with the temperature on the cosmological horizon of a FRW universe.

The organization of the paper is as follows. In Sec. II, we first obtain the cosmological horizon in a FRW universe, and then use the Damour- Ruffini method to obtain its Hawking

radiation. Sec. III is devoted to the conclusion and discussion.

II.   THE COSMOLOGICAL HORIZON AND ITS HAWKING RADIATION IN A FRW UNIVERSE

The metric of a FRW universe is

$$ds^2 = -dt^2 + a^2(t)\left(\frac{d\rho^2}{1-k\rho^2} + \rho^2 d\Omega_2^2\right), \tag{2.1}$$

where t is the cosmic time, ρ is the comoving radial coordinate, a is the scale factor, $d\Omega_2^2$ denotes the line element of a 2-dimensional sphere with unit radius, $k = 1$, 0 and -1 represent a closed, flat and open FRW universe respectively.

For the convenience, we define $r = a\rho$. Therefore, the metric (2.1) can be rewritten as

$$ds^2 = -\frac{1-r^2/r_A^2}{1-kr^2/a^2}dt^2 - \frac{2Hr}{1-kr^2/a^2}dtdr + \frac{1}{1-kr^2/a^2}dr^2 + r^2 d\Omega_2^2, \tag{2.2}$$

where $r_A = 1/\sqrt{H^2 + k/a^2}$ is the location of apparent horizon in a FRW universe.

Note that the metric of the de Sitter spacetime is

$$ds^2 = -\left(1-\frac{r^2}{l^2}\right)dt^2 + \left(1-\frac{r^2}{l^2}\right)^{-1}dr^2 + r^2 d\Omega_2^2. \tag{2.3}$$

and the FRW metric (2.2) can become

$$ds^2 = -\frac{1-r^2/r_A^2}{1-kr^2/a^2}(dt + \frac{Hr}{1-r^2/r_A^2}dr)^2 + \frac{1}{1-r^2/r_A^2}dr^2 + r^2 d\Omega_2^2. \tag{2.4}$$

Therefore, it can be easily found that the de Sitter spacetime is just a special case of the FRW universe where $k = 0$ and $r_A = H^{-1} = l$ is a constant in (2.4). On the other hand, we know that $r = l$ is the cosmological horizon of the de Sitter spacetime, therefore, there may be a corresponding cosmological horizon in a FRW universe. By using the null property of the cosmological horizon and the spherical symmetry in (2.2), we can indeed obtain that the corresponding cosmological horizon $r = r_H(t)$ which satisfies

$$g^{\mu\nu}\frac{\partial f}{\partial x^\mu}\frac{\partial f}{\partial x^\nu} = 0, \tag{2.5}$$

is

$$1 - r_H^2/r_A^2 = \dot{r}_H^2 - 2Hr_H\dot{r}_H. \tag{2.6}$$

where $f = r - r_H(t)$. From (2.6), it can be also easily checked that the corresponding cosmological horizon $r_H(t)$ is just the cosmological horizon of the de Sitter spacetime when $k = 0$ and $\dot{r}_H = 0$.

In the following, we investigate the Hawking temperature of the corresponding cosmological horizon $r = r_H(t)$ in a FRW universe. For the simplicity, we just consider the Klein-Gordon field in a FRW universe. And the Klein-Gordon equation

$$(\Box - m^2)\Phi = \frac{1}{\sqrt{-g}}\frac{\partial}{\partial x^\mu}(\sqrt{-g}g^{\mu\nu}\frac{\partial}{\partial x^v})\Phi - m^2\Phi = 0. \tag{2.7}$$

can be rewritten in the FRW coordinates (2.2) such that

$$-\frac{\partial}{\partial t}(\frac{1}{\sqrt{1-\frac{k}{a^2}r^2}}\frac{\partial}{\partial t})\frac{\rho(t,r)}{r} - \frac{\partial}{\partial t}(\frac{Hr}{\sqrt{1-\frac{k}{a^2}r^2}}\frac{\partial}{\partial r})\frac{\rho(t,r)}{r} - \frac{1}{r^2}\frac{\partial}{\partial r}(\frac{r^2}{\sqrt{1-\frac{k}{a^2}r^2}}Hr\frac{\partial}{\partial t})\frac{\rho(t,r)}{r}$$

$$+ \frac{1}{r^2}\frac{\partial}{\partial r}[\frac{r^2}{\sqrt{1-\frac{k}{a^2}r^2}}(1-r^2/r_A^2)\frac{\partial}{\partial r}]\frac{\rho(t,r)}{r} = [m^2 + \frac{l(l+1)}{r^2}]\frac{1}{\sqrt{1-\frac{k}{a^2}r^2}}\frac{\rho(t,r)}{r}, \tag{2.8}$$

$$\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta\frac{\partial}{\partial\theta})Y_{lm}(\theta,\varphi) + \frac{1}{\sin^2\theta}\frac{\partial^2}{\partial\varphi^2}Y_{lm}(\theta,\varphi) + l(l+1)Y_{lm}(\theta,\varphi) = 0, \tag{2.9}$$

where m is the rest mass of the Klein-Gordon particle, $Y_{lm}(\theta,\varphi)$ is the usual spherical harmonics and Φ has been separated as

$$\Phi = \frac{1}{r}\rho(t,r)Y_{lm}(\theta,\varphi). \tag{2.10}$$

In order to investigate the behavior of the scalar field near the cosmological horizon, we introduce the generalized tortoise coordinate transformation

$$r_* = r + \frac{1}{2\kappa}\ln[r_H(t) - r],$$
$$t_* = t - t_0, \tag{2.11}$$

where κ is an adjustable constant, $r_H(t)$ is just the location of the cosmological horizon, and t_0 is a constant representing the time when the particles are radiated from the horizon. Note that, κ can be just the surface gravity of the event horizon or cosmological horizon in the stationary spacetimes.

From (2.11), the radial equation (2.8) becomes

$$\{-\frac{2\kappa(r-r_H)(\dot{a}^2+k+a\ddot{a})}{a[r(2r\kappa-2r_H\kappa+1)\dot{a}-a\dot{r}_H]}+\frac{2\left(l^2+l+m^2r^2\right)\kappa a(r-r_H)}{r^2[r(2r\kappa-2r_H\kappa+1)\dot{a}-a\dot{r}_H]}\}\rho$$

$$+\{\frac{-[\dot{r}_H^2+(r-r_H)\ddot{r}_H-1]a^2+[(r+r_H)\dot{a}\dot{r}_H+r(r-r_H)(2r\kappa-2r_H\kappa+1)\ddot{a}]a}{a(r-r_H)[r(2r\kappa-2r_H\kappa+1)\dot{a}-a\dot{r}_H]}$$

$$+\frac{r[2\kappa r^2+2\kappa r_H^2-(4r\kappa+1)r_H](\dot{a}^2+k)}{a(r-r_H)[r(2r\kappa-2r_H\kappa+1)\dot{a}-a\dot{r}_H]}\}\frac{\partial\rho}{\partial r_*}+\{\frac{(2r\kappa-2r_H\kappa+1)^2(\dot{a}^2+k)r^2}{2\kappa a(r-r_H)[r(2r\kappa-2r_H\kappa+1)\dot{a}-a\dot{r}_H]}$$

$$+\frac{-2(2r\kappa-2r_H\kappa+1)\dot{a}\dot{r}_H ar+a^2[-(2r\kappa+1)^2+4\kappa r_H(2r\kappa+1)-4\kappa^2r_H^2+\dot{r}_H^2]}{2\kappa a(r-r_H)[r(2r\kappa-2r_H\kappa+1)\dot{a}-a\dot{r}_H]}\}\frac{\partial^2\rho}{\partial r_*^2}$$

$$+\frac{2\kappa(r-r_H)\dot{a}}{r(2r\kappa-2r_H\kappa+1)\dot{a}-a\dot{r}_H}\frac{\partial\rho}{\partial t_*}+2\frac{\partial^2\rho}{\partial t_*\partial r_*}+\frac{2\kappa a(r-r_H)}{r(2r\kappa-2r_H\kappa+1)\dot{a}-a\dot{r}_H}\frac{\partial^2\rho}{\partial t_*^2}=0. \quad (2.12)$$

when $r\to r_H$ and $t\to t_0$, the radial equation (2.12) is

$$A\frac{\partial^2\rho}{\partial r_*^2}+2\frac{\partial^2\rho}{\partial t_*\partial r_*}+\alpha_0\frac{\partial\rho}{\partial r_*}=0, \quad (2.13)$$

where we have used the equation (2.6) and

$$A=-\frac{H\dot{r}_H-(H^2+k/a^2)r_H}{\kappa(Hr_H-\dot{r}_H)}+2\dot{r}_H,\quad \alpha_0=\frac{(H^2+k/a^2)r_H-H\dot{r}_H+\ddot{r}_H-\frac{\ddot{a}}{a}r_H}{\dot{r}_H-Hr_H}. \quad (2.14)$$

The two linearly independent solutions of (2.13) are

$$\rho_{out}=e^{-i\omega t_*}, \quad (2.15)$$

and

$$\rho_{in}=e^{-i\omega t_*+2i\omega r_*/A}e^{-\alpha_0 r_*/A}. \quad (2.16)$$

which is just inside the cosmological horizon ($r<r_H$). By using the fact that usually the Klein-Gordon equation in the tortoise coordinates can be reduced to the standard form of wave equation near the horizon [13, 14, 39, 40]

$$\frac{\partial^2\rho}{\partial r_*^2}+2\frac{\partial^2\rho}{\partial t_*\partial r_*}=0, \quad (2.17)$$

we can adjust the parameter κ to make $A=1$, and

$$\kappa=\frac{H\dot{r}_H-(H^2+k/a^2)r_H}{(Hr_H-\dot{r}_H)(2\dot{r}_H-1)}. \quad (2.18)$$

Note that, $A=1$ can also be implied from the special case, that of the de Sitter spacetime. In this special case, $k=0$ and $\dot{r}_H=0$ with $r_A=H^{-1}=l$, the κ in (2.18) is $\kappa=1/l$ which is

just the surface gravity of the cosmological horizon in the de Sitter spacetime. In addition, from (2.18), it can also be found that κ is indeed a constant just related to t_0.

Therefore, the ingoing wave of the Klein-Gordon filed near the cosmological horizon can be further rewritten as

$$\rho_{in} = Ce^{-i\omega t_* + 2i\omega r_*}e^{-\alpha_0 r_*} = Ce^{-i\omega t_*}e^{2i\omega r - \alpha_0 r}(r_H - r)^{i\omega/\kappa - \alpha_0/2\kappa}. \tag{2.19}$$

where we have used (2.11) and added the normalized factor C. Note that, ρ_{out} represents an outgoing wave and is well-behaved when analytically extended outside $r > r_H$. However, we can find that the ingoing wave ρ_{in} (2.19) has a logarithmic singularity at the cosmological horizon $r = r_H$ and is not analytical on the cosmological horizon. Thus we can extend it by analytical continuation from the inside to outside of the cosmological horizon [13, 14, 39–41]

$$(r_H - r) \to |r_H - r|e^{i\pi} = (r - r_H)e^{i\pi}, \tag{2.20}$$

and then the ingoing wave (2.19) becomes

$$\tilde{\rho}_{in} = Ce^{-i\omega t_*}e^{2i\omega r - \alpha_0 r}(r - r_H)^{i\omega/\kappa - \alpha_0/2\kappa}e^{-\frac{i\pi\alpha_0}{2\kappa}}e^{-\frac{\pi\omega}{\kappa}} = Ce^{-i\omega t_* + 2i\omega r_*}e^{-\alpha_0 r_*}e^{-\frac{i\pi\alpha_0}{2\kappa}}e^{-\frac{\pi\omega}{\kappa}}, r > r_H. \tag{2.21}$$

By using the Heaviside function Y

$$Y(x) = \begin{cases} 1, & x \geq 0 \\ 0, & x < 0 \end{cases} \tag{2.22}$$

the complete ingoing wave can be

$$\phi_\omega^{in} = N_\omega[Y(r_H - r)\rho_{in} + Y(r_H - r)\tilde{\rho}_{in}]. \tag{2.23}$$

where N_w is a normalization factor. Physically, the waves (2.19) (2.21) can represent an ingoing particle wave inside the cosmological horizon and an outgoing antiparticle wave of negative energy outside the cosmological horizon [39]. Another interpretation is that an antiparticle of positive energy ingoing in the past being scattered forward in time at the cosmological horizon, and the ingoing wave describing this antiparticle state is just (2.23). Therefore, similar to the WKB approximation in the quantum mechanical barrier penetration, N_ω^2 can represent the strength of a particle wave ingoing or tunneling from the cosmological horizon. More details, the antiparticle state ϕ_ω^{in} is split into two components, a particle wave of strength N_ω^2 ingoing from the horizon and a negative-energy flux of antiparticles N_ω^2 outgoing in the future toward the outside of the cosmological horizon. The

latter can always be interpreted as an antiparticle wave of strength $N_\omega^2 e^{-2\pi\omega/\kappa}$ with positive energy flux ingoing in the past from the outside of the cosmological horizon [39].

In the following, we use a simple argument to obtain the temperature of radiation [13, 14, 39, 40]. As ρ_{in} is already normalized, the scalar product of ϕ_ω^{in} in (2.23) is

$$(\phi_{\omega_1}^{in}, \phi_{\omega_2}^{in}) = N_{\omega_1} N_{\omega_2} (\delta_{\omega_1\omega_2} - e^{-\pi(\omega_1+\omega_2)/\kappa} \delta_{\omega_1\omega_2}), \tag{2.24}$$

where we have used the fact that the inner product of the wave function is normalized to minus δ function for the Bose particle with negative energy. Note that, if $\kappa < 0$ in (2.24), we obtain

$$(\phi_w^{in}, \phi_w^{in}) = -1 = N_\omega^2 (1 - e^{-2\pi\omega/\kappa}). \tag{2.25}$$

which is just a thermal spectrum with a temperature $T = -\kappa/2\pi$. While if $\kappa > 0$, we obtain

$$(\phi_\omega^{in}, \phi_\omega^{in}) = 1 = N_\omega^2 (1 - e^{-2\pi\omega/\kappa}). \tag{2.26}$$

which is apparently not a thermal spectrum. However, we can redefine the complete ingoing wave in (2.23) as

$$\phi_\omega^{in'} = e^{\frac{\pi\omega}{\kappa}} N_\omega [Y(r_H - r)\rho_{in} + Y(r_H - r)\tilde\rho_{in}], \tag{2.27}$$

from which we obtain

$$(\phi_\omega^{in'}, \phi_\omega^{in'}) = 1 = N_\omega^2 (e^{2\pi\omega/\kappa} - 1). \tag{2.28}$$

which is a thermal spectrum with the temperature $T = \kappa/2\pi$.

In other words, we obtain the thermal spectrum in both cases

$$N_\omega^2 = 1/[\exp(\omega/K_B T) - 1], \tag{2.29}$$

and the temperature T is

$$T = \frac{|\kappa|}{2\pi} = |\frac{(H^2 + k/a^2)r_H - H\dot r_H}{2\pi(Hr_H - \dot r_H)(2\dot r_H - 1)}|. \tag{2.30}$$

III. CONCLUSION AND DISCUSSION

Whether there is a Hawking radiation in a FRW universe is a very interesting question. From the fact that the de Sitter spacetime can be a special case of a FRW universe and there is a Hawking radiation from the cosmological horizon of the de Sitter spacetime, therefore, it may also have a corresponding Hawking radiation in a FRW universe. Indeed, there have

been some clues showing that there is a Hawking radiation from the apparent horizon in a FRW universe. However, in our paper, after finding the corresponding cosmological horizon of a FRW universe first, and then investigating the behavior of a Klein-Gordon field near the cosmological horizon, we obtain that the Hawking radiation comes from the cosmological horizon of a FRW universe. Note that, when $\dot{r}_H = 0$, we can see that the cosmological horizon in (2.6) is same with the apparent horizon. And the temperature in (2.30) is

$$T = \frac{1}{2\pi H r_A^2},$$

(3.1)

which is apparently not same as the temperature $T = \frac{1}{2\pi r_A}$ in some previous results from the apparent horizon [31, 32] . However, there the temperature $T = \frac{1}{2\pi r_A}$ is measured by the Kodama observer. From which, the temperature measured by the observer $(\partial/\partial t)^a$ in (2.2) is $T = \frac{1}{2\pi H r_A^2}$ [31]. Furthermore, $\dot{r}_H = 0$ ensures the observer in the coordinates system in (2.11) same as the observer $(\partial/\partial t)^a$ in (2.2). Therefore, our result under the condition $\dot{r}_H = 0$ is in fact consistent with the result in reference [31, 32]. In addition, we can further find that this condition $\dot{r}_H = 0$ is consistent with the underlying integrable condition in [31, 32]. From $\dot{r}_H = 0$, we can find that the cosmological horizon and apparent horizon are same. Therefore, $\dot{r}_H = 0$ can reduce $\dot{r}_A = 0$. On the other hand, from equations (7) and (10) in [31], the underlying integrable condition coming from $\partial_{\tilde{r}}\partial_t S = \partial_t\partial_{\tilde{r}}S$ can also deduce $\dot{r}_A = 0$. All these consistences partly support the validity of our temperature (2.30).

It should be emphasized that our temperature (2.30) is also valid just under some conditions (like the quasi-static or adiabatic condition), which can be implicated from the limits $r \to r_H$ and $t \to t_0$ in (2.13). Therefore, it would be very interesting to have further research on the validity of this temperature (2.30) to obtain the explicit conditions. In addition, the temperature from the apparent horizon is obtained by using the Hamilton-Jacobi equation, and the Hamilton-Jabobi equation can be a WKB approximation solution of the Klein-Gordon equation. Thus it is also an interesting work to research the approximation condition from the Klein-Gordon equation to the Hamilton-Jabobi equation. Moreover, from the modern quantum field theory, the temperature comes from two different vacuums [42], and there have been some works to show the two corresponding different vacuums in the Hamilton-Jacobi method, therefore, the two underling different vacuums in the Damour-Ruffini method are also interesting to find out [43–45]. In addition, it should be noted that there is a different approach of particle production in a FRW universe named Parker parti-

cle production [46]. Apparently, the particle production in our paper is different from the Parker particle production for two reasons. First, the spectrums of numbers of particles in Parker particle production are usually not an absolutely thermal spectrum of black body. Second, the two different vacuums constructed are apparently different [43–45]. Therefore, it would be very interesting to give further study on the underlying relationship between the particle production in our paper and the Parker particle production.

IV. ACKNOWLEDGEMENTS

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